**19.09.2020 FN SESSION a b c**

1) a + b + c = 13 what is the maximum value of (a – 3) (b – 2) (c + 1)?

a) 26 b) 27 c) 30 d) 19

Soln :

***a + b + c = 9 🡪 abc will be maximum when a=b=c.***

***7 + 1 + 1 = 9 🡪 abc = 7 \* 1 \* 1 = 7***

***5 + 3 + 1 = 9 🡪 abc = 5 \* 3 \* 1 = 15***

***4 + 2 +3 = 9 🡪 abc = 4 \* 2 \* 3 = 24***

***3 + 3 + 3 = 9 🡪 abc = 3 \* 3 \* 3 = 27***

*a + b + c = 13*

*a \* b \* c will be maximum when ?*

*a = b = c*

***a + b + c = 13***

(a – 3) + (b – 2) + (c + 1) = a + b + c - 3 - 2 + 1 =13 – 3 – 2 + 1 = 9

(a – 3)+ (b – 2)+ (c + 1) = 9

a-3 = b-2 = c+1 = k

(a – 3)+ (b – 2)+ (c + 1)

(a – 3)+ (b – 2)+ (c + 1) = 9

k + k + k = 9

k = 3.

(a – 3) (b – 2) (c + 1) = 3 \* 3 \* 3 = 27.

2) If abcd = 81 what is the minimum value of a + b + c + d?

a) 18 b) 9 c) 12 d) 36

a + b + c = 13

a b c will be maximum when a = b = c

a\*b\*c\*d =81

Minimum value of a + b + c + d when a = b = c = d

a = b = c =d = k

k \* k \* k \* k = 81

k = 3.

a + b + c + d = 3 + 3 + 3 + 3 = 12

if a \* b \* c \* d = 81 when will we have the minimum value of a + b + c + d

we will have the minimum value of a + b + c + d when a = b = c = d

a \* b \* c \* d = 81 when a = b = c = d

a4 = 81

a = 3

a + b + c + d = 3 + 3 + 3 + 3 = 12.

3) If x + y= 25 and x2 y 3+ y2 x3 = 25, what is the value of xy?

a) +0 or -0 b) + 1 or -1 c) +5 or -5 d) +4 or -4

Soln:

x2 y 3+ y2 x3 = 25

x2y2(y + x ) = 25

x2y2(25 ) = 25

x2y2= 25 / 25

x2y2= 1

xy = +1 or -1

4) If x and y are both positive, then the minimum value of (x + y) (1/x + 1/y) is:

a) 0 b) 1 c) 2 d) 4

Soln :

x > 0 & y > 0

assume x=1 & y=1

(x + y)(1/x + 1/y) = (2)(2) = 4 minimum value

assume x=2 & y=1

(x + y)(1/x + 1/y) =(3)(3 / 2) = 9 / 2

Assume x = 1 / 2 & y = 1 / 2

( 1 / 2 +1/2)(2 + 2) = 4

5) If a, b, c and d are four positive number such that a + b + c + d = 4, then what is the maximum value of (a +1) (b + 1) (c + 1) (d + 1)?

a) 32 b) 8 c) 16 d) 81

a + b + c + d = 4

a \* b \* c \* d will be maximum when a = b = c =d

(a +1)\* (b + 1)\* (c + 1)\* (d + 1) will have max val when a+1 = b+1 = c+1 = d+1 = k

(a +1)+(b + 1)+ (c + 1)+ (d + 1)= a + b + c + d + 1 + 1 + 1 + 1= 4 + 4 = 8

k + k + k + k = 8

4 k = 8

k = 2

(a +1)\* (b + 1)\* (c + 1)\* (d + 1) = 2 \* 2 \* 2 \* 2 = 16.

Soln :

a + b + c + d = 4

a \* b \* c \* d will be maximum when a = b = c = d

(a +1) (b + 1) (c + 1) (d + 1)

(a +1) + (b + 1) + (c + 1) + (d + 1) =a + b + c +d + 1 + 1 + 1 + 1 = 4 + 4 = 8

(a +1) + (b + 1) + (c + 1) + (d + 1) = 8

Maximum value for (a +1) (b + 1) (c + 1) (d + 1) is when a+1 = b+1 = c+1 = d+1 = k

k + k + k + k = 8

4k = 8

k = 2

(a +1) (b + 1) (c + 1) (d + 1) = 2 \* 2 \* 2 \* 2 = 16.

6) If x = 7 + 4√3 and xy = 1, then the value of (1/x2 + 1/y2) is:

a) 194 b) 57 c) 85√3 d) 7 + 4√3/ 7 - 4√3

Soln :

(1/x2 + 1/y2) = y2 + x2 / x2y2

xy = 1 🡪 y = 1 / x

x = 7 + 4√3

y = (1 / 7 + 4 √3) \* (7 - 4 √3 / 7 - 4 √3) = (7 - 4 √3) / ( 49 – 48 ) = (7 - 4 √3) = y

(a+b) \* (a-b) = a2 – b2

A= 7 ; b = 4 √3

y2 + x2 / x2y2 = y2 + x2  / 1 = (7 + 4√3) 2 + (7 - 4√3) 2 = 49 +48 +2 \*7 \* 4√3 +49 +48 -2 \*7 \* 4√3 = 194

1/x2 + 1/y2 = y2 + x2 / x2y2

x = 7 + 4√3

xy = 1 🡪 y = 1 / x

y = (1 / 7 + 4√3) \* (7 - 4√3 / 7 - 4√3) = (7 - 4√3 ) / (7)2 – (4√3)2 =(7 - 4√3 ) / 49 – (16 \* 3) =(7 - 4√3 )

y =(7 - 4√3 )

(7 + 4√3)2 + (7 - 4√3 )2 / (7 + 4√3)2 (7 - 4√3 )2

(7 + 4√3)2 + (7 - 4√3 )2 / [(7 + 4√3) (7 - 4√3 )]2

a2 + b2 = (a+b)2 -2ab

Numerator.

(7 + 4√3 + 7 - 4√3)2 – 2 (7 + 4√3 ) (7 - 4√3 ) = 196 – 2 (49 – 48 ) = 194 🡪Numerator

Denominator

[(7 + 4√3) (7 - 4√3 )]2 = (7)2 – (4√3)2 = 49 – 48 = 1

Ans 🡪 194 / 1 = 194

8) If a, b, c are positive real number then least value of (a + b + c) (1/a + 1/b + 1/c):

a) 1 b) 9 c) 12 d) none of these

Soln :

ARITHEMETIC MEAN = AVERAGE = (a + b + c) / 3

GEOMETRIC MEAN FOR a, b = Sq rt of (ab)

GEOMETRIC MEAN FOR a, b , c = Cube rt of (abc)

GEOMETRIC MEAN FOR a, b , c , d = 4th rt of (abcd)

AM >= GM

(a + b + c) / 3 >= (abc)1/3 🡪 Sq rt (abc) = (abc)1/2

1 / 3 (a + b + c) >= (abc)1/3

1 / 3(1 / a + 1 / b + 1 /c) >= (1 / a \* 1 / b \* 1 / c)1 / 3

1 / 3(1 / a + 1 / b + 1 /c) >= (1 / abc)1 / 3

log am = m log a

log ab = log a + log b

log (abc)1/3 = 1/3 log abc = 1/3 (log a + log b + log c)

log (abc)1/3 = 1/3 (log a + log b + log c)

log (abc)1/3 = 1/3 log(a + b + c)

(abc)1/3 = 1/3 (a + b + c)

1 / 3 (1 / a + 1 / b + 1 /c)1 / 3 ( a + b + c ) >= (abc)1/3 (1 / abc )1 / 3

>= (abc)1/3 \* 1 / (abc)1 / 3 =1 = RHS

(1 / a + 1 / b + 1 /c) ( a + b + c ) > = 9

a = b = c = 1

( 1 + 1 + 1 ) ( 1 + 1 + 1 ) = 3 \* 3 = 9

9) If a > 0, b > 0 and ab = 1, then the least value of expression (1 + a) (1 + b) is:

a) 2 b) 4 c) 1 d) 1 / 4

a = 1,b=1 🡪 2 \* 2 = 4

Soln :

If ab = 1, then a + b takes minimum or least value when a=b

Therefore a= b = 1.

least value of expression (1 + a) (1 + b) = (1 + 1) (1 + 1) = 4

10) If a, b, c are all positive and not equal then the value of

(a + b +c) (ab + bc + ca) is:

abc

a) Less than 9 b) greater than 9 c) Less than or equal to 9 d) can’t be determined

Soln :

(a + b +c) (ab + bc + ca) / abc is:

(a + b +c) (ab + bc + ca) / abc = (a + b +c) ( 1 / c +1 / a + 1 / b ) >= 9

11)If a, b, c are all positive, then the minimum value of the expression

(a2 + a +1) ( b2 + b + 1)(c2 + c + 1) is:

Abc

(1 + 1 + 1 ) ( 1 + 1 + 1) ( 1 + 1 + 1) / 1 = 3 \* 3 \* 3 = 27.

a) 3 b) 9 c) 27 d) 1

12) If 1 <= x <= 3 and 2 <= y <= 4, what is the maximum value of (x/y)

a) 2/3 b) 4 c) 3/2 d) 2

Soln :

A fraction will have maximum value when numerator is as large as possible and denominator is small as possible

Max ( x / y) =Max (x) / Min (y) = 3 / 2

13) If a + b + c = 3 , a2 + b2 + c2 = 6 and 1/a + 1/b + 1/c=1, where a, b, c are all non-zero, then abc is:

a) 1/3 b) 2/3 c) 3/2 d) 1

(a + b + c ) 2 = a2 + b2 + c2 + 2 (ab + bc + ac )

1/a + 1/b + 1/c=1

(bc + ac +ab / abc ) = 1

( bc + ac + ab ) = abc

(a + b + c ) 2 = a2 + b2 + c2 + 2 (ab + bc + ac )

(3)2  = 6 + 2 (abc)

9 – 6 = 2 abc

3 = 2 abc

3 / 2 = abc

Soln :

1/a + 1/b + 1/c=1

bc + ac + ab / abc = 1

bc + ac + ab = abc

(a + b + c )2 = a2 + b2 + c 2 + 2(ab + bc + ca)

(a + b + c )2 = a2 + b2 + c 2 + 2(abc)

(3)2 = 6 + 2 abc

9 – 6 = 2abc

3 / 2 = abc

14. What is true about x, if x + b2 = a2 and x is a prime number? (a , b > 0)

x = a2 – b2

5 = 32 – 22

7 = 42 – 32

11 = 62 – 52

x = a + b

1) In a right angled triangle ∠B and ∠A are acute angles. If ∠B and ∠A are acute angles. If ∠A - ∠B = k, where A and B are integers, then how many integers values can k take?

a) 80 b) 88 c) 45 d) 89

A

90

C B

A + B = 90

∠A - ∠B = k,

89 – 1 = 88 = k

88 – 2 = 86

87 – 3 = 84

86 – 4 = 82

46 – 44 = 2

45 – 45 = 0

44 – 46 = -2

1 – 89 = -88

2 – 88 = 44 values

0 = 1 value

-2 -88 = 44 values

89 – 1 = 88 85 – 5 = 80 44-46 = - 2

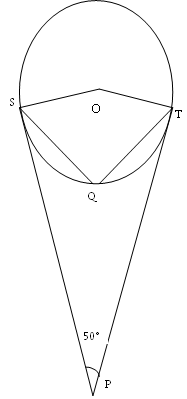
88 – 2 = 86 78 43 – 47 = -4

87 – 3 = 84 46 – 44 = 2

86 – 4 = 82 45 – 45= 0 1 – 89 = -88

88 , 86 , 84 , 82 , 80…….,2, 0 , - 2, - 4, -6, - 8 ……..-88= 44 + 1 + 44= 89

2) In the given figure ‘O’ is the centre of the circle SP and TP are the two tangents at S and T respectively, ∠SPT is 50°, the value of ∠SQT is:



65

??

PSOT IS A QUADRILATERAL (4 SIDES)

Total angles inside a quadrilateral = 360

Angles P + O + S+ T = 360

P + O = 360 – 180 = 180 (S + T = 180)

50 + O = 180  O = 130 degrees

Arc ST subtends an angle of 130 degrees at the centre.

SRT = 1 / 2 SOT

= 1 / 2 \* 130 = 65 degrees

R + Q = 180  Q = 180 – 65 = 115.

a) 125° b) 65° c) 115° d) none of these

3) A ladder 6.5m long is standing against a wall and the difference between the base of the ladder and the wall is 5.2m. If the top of the ladder now slips by 1.4m, then by how much will the foot of the ladder slip?

a) 1.2m b) 0.8m c) 0.75m d) none

Soln :

C

1.4m

P 6.5 m

3.9 m 6.5m

A 5.2 m B Q

BC ,PQ = 6.5m AB = 5.2 m CP = 1.4 m BQ = ?

TRI ABC

BC2 = AB 2 + AC 2 = (5.2)2 + AC2 = (6.5)2

AC2 = 42.25 – 27.04 = 15.21 🡪AC = 3.9 m

39 \* 39 = 1521

40 \* 40 = 1600

Heigt of the wall is 3. 9 m

AC- CP = AP

3.9 – 1.4 = 2.5 m

AQ = AB + BQ

In TRI APQ

PQ2 = AQ2 + AP2

(6.5)2 –(2.5)2 = 42.25 – 6.25 = 36 = AQ2

AQ = 6

AQ = AB +BQ = 5.2 + x = 6 🡪 x =0.8 m

AC2 = BC2 – AB2

= 42.25 – 27.04 = 15.21

AC = √15.21 = 3.9 m

39 \* 39 = 1521

AP = 3.9 – 1.4 = 2.5

PQ2 = AQ2 + AP2

PQ2 – AP2 = AQ2

42.25 – 6.25 = AQ2

36 =AQ2  AQ = 6

BQ Length that the foot of the ladder slipped= AQ – AB = 6 – 5.2 = 0.8 m

4) In a triangle all the three angles A,B, C are integers, then the number of values that A,B and C can take is

a) 89 b) 90 c) 178 d) 180

A + B + C = 180

1 + 178 + 1

2 + 177 + 1

3 + 176 + 1

4 + 175 + 1

.

.

.

.

178 + 1 + 1

A CAN TAKE VALUES 1-178

B CAN TAKE VALUES 1-178

C CAN TAKE VALUES 1-178

A + B + C = 180

1 + 1 + 178

1 + 178 + 1

178 + 1 + 1

Range of A, B, C is 1 to 178.

5) Two trains Punjab mail and Lucknow mail starts simultaneously from patiyala and Lakhimpur respectively towards each other with the speed of 40km/hr and 60km/hr respectively on the same track Lakhimpur is 500km due east of Patiyala. A plane starts flying at 200 km/hr at the same time from Patiyala to jalandhar. Jalandhar is 100km due north of Patiyala. After travelling sometime two trains Punjab mail and Lucknow mail collides with each other. The plane moves continuously to and fro between Patiyala to Jalandhar till the collision of the trains. How far would the plane have travelled?

a) 100km b) 1000km c) 2000km d) can’t be determined

Jala 100 300 500 700 900 1100

100 km

200 km /hr

Flight 200 400 600 800 1000

Patiala -Punjab mail 500 km Lucknow Mail-Lakhimpur

40 km / hr 60 km / hr

200 km in 5hrs 300 km in 5hrs

Collision Point

Time taken = Dis / Speed = 500 / 100 = 5 hrs

Distance taken covered by the flight until collision is = 200 \* 5 = 1000 km

6) In the above question (number 10) what is the distance between the place of accident and the plane at the moment of accident of two trains?

a) 200 km b) 250 km

c) 400 km d) can’t be determined

Collision happens at 200 km from Patiala

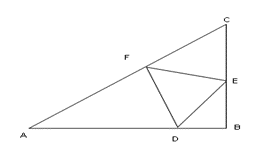
At the time of Collision the fligt would have travelled 1000 km.

Distance between trains and flight at the the moment of collision.

At the end of 5 hrs Flight will be in Patiala at point P

Collision happens when P travels 200km & L travels 300km

7) In the given figure ∠B is right angle. AD: BD = 3:2 and CE: BE = 5:2 and AF: FC=1:1. What is the area of ΔABC, if the area of ΔBDE is 20 cm2?



1

1

3

5

2

2

Tri BDE area =20 cm2

Area of BDE = 1 / 2 \* BD \* BE

AB = 5 parts

BD = 2 parts

BD = 2 / 5 AB

BC = 7 parts

BE = 2 parts

BE = 2 / 7 BC

Area of BDE = 1 / 2 \* BD \* BE

= 1 / 2 \* 2 / 5 AB \* 2 / 7 BC

20 = 4 / 35 \*(1/ 2) \* AB \* BC

20 \* 35 / 4 = Area of TRI ABC

175 cm2 = Area of TRI ABC

AB = 5 parts  BD = 2parts

= 1 / 2 \* 2 /5 AB \* 2 / 7 BC

= 4 /70 \* AB \* BC

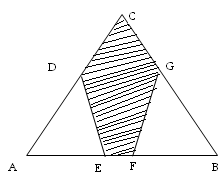
20 = 4 / 35 \* (1 / 2) AB \* BC

20 \* 35 / 4 = Area of ABC

175 m2 = Area of ABC

a) 40 cm2 b) 35 cm2 c) 52.5 cm2 d) none

8) In the given figure ABC is a triangle in which CDEFG is pentagon. Triangle ADE and BFG are equilateral triangles each with side 2cm and EF = 2cm. Find the area of the pentagon:



60

60

60

AD = DE = AE = 2cm

GF = FB = BG =2cm

EF = 2cm

Equilateral triangle Area = root3 / 4 \* a2

ADE is equilateral triangle

A = 60 deg , D , E = 60 deg

B = 60 deg ,F & G = 60 deg

As C is also 60 deg ABC is an equilateral triangle.

All sides are equal in tri ABC and it is 6 cm

Area of Pentagon = Area of Large Triangle – Area of the small traingles

= root3 / 4 ( 36 – ( 4 + 4 ))

= root3 / 4 \* 28 = root 3 \* 7

Angle A & B is 60 degrees

Angle C will also 60 degrees

We can say ABC is a equilateral Traiangle

AE = 2 cm ; BF = 2cm ; EF = 2cm

AB = AE +BF +EF = 2 + 2 + 2 = 6 cm

Area of Equilateral triangle = √3 / 4 \* a2

Area of Pentagon = Area of ABC – (Area of ADE + Area of BFG)

= √3 / 4 \* 62 – ( √3 / 4 \* 22 + √3 / 4 \* 22)

a) 8√3 cm2 b) 7√3 cm2 c) 15√3 cm2 d) 11.28 cm2

10) ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. ∠ADP = ∠BEQ = 100°. What is the value of ∠PRD?

a) 60° b) 50° c) 20° d) can’t be determined

11) In the above question if OC is the half of the AB, then the value of ∠ACB is:

a) 60 degree b) 90 degree c) 80 degree d) can’t be determined

x =a2 - b2

(x) 7 = (a)42 – (b)32 = 16 – 9 = 7 🡪Prime no

(x)5 = (a) 32 – (b) 22 = 9 – 4 = 5 🡪 Prime no

x = a +b